Geometric Control Theory: Recent Advances and Lie Group Applications

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Abstract

Geometric or mathematical control theory, a branch of applied mathematics, investigates the principles and methods for analysing and constructing control systems through a differential geometric and topological framework. This talk will introduce the core concepts of the field, emphasising control-affine systems on smooth manifolds as a prominent class of systems studied in recent years. Such systems are defined on a smooth manifold M with dynamics governed by:

$$\Sigma_M: \quad \dot{x}(t) = f_0(x(t)) + \sum_{j=1}^{m} \omega_j(t) f_j(x(t)), \quad \omega \in \mathcal{U},$$

where $f_0, f_1, \ldots, f_m$ are smooth vector fields defined on M and $\omega = (\omega_1, \ldots, \omega_m)$ is a piecewise constant control taking values in a compact convex set $\Omega \subset \mathbb{R}^m$ with $0 \in \text{int}\ \Omega$. The talk will then explore recent trends in geometric control theory, in particular the evolution of research towards problems involving symmetry, algebraic structures and geometric reduction. Key challenges in this area include characterising the controllability of systems with nonlinear geometries, and using Lie-theoretic tools to simplify complex dynamics. Current work often focuses on systems defined on Lie groups or homogeneous spaces, where symmetries allow tractable analysis and design.

To illustrate these ideas, the talk will conclude with examples on 2-step nilpotent Lie groups, highlighting how their simplified algebraic structure and geometric properties provide a natural setting for studying controllability. This case study aims to demonstrate the interplay between geometric control theory, Lie group structures, and real-world applications, and to provide a concrete perspective on modern methodological advances in the field.

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