New covering arrays of strength 4 and q symbols from three truncated Möbius planes in $PG(3, q)$, for odd prime power q

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(Joint work with Kianoosh Shokri and Brett Stevens)

In this talk, we will discuss recent work connecting finite fields, combinatorial designs and finite geometry. A strength-t covering array of size N, denoted by $\mathrm{CA}(N; t, k, v)$, is an $N \times k$ array over a v-set of symbols such that for any t-set of columns, each t-tuple occurs at least once in a row. Raaphorst et al. [3] construct a $\mathrm{CA}(2q^3 - 1; 3, q^2 + q + 1, q)$ from two projective planes, $\mathrm{PG}(2, q)$, on the same set of points such that any line in one plane intersects any line in the other plane in at most 2 points. In [2], Colbourn et al. call two such projective planes "orthogoval"; they study sets of mutually orthogoval projective and affine planes, and discuss their connections to covering arrays.

Our present work extends the result by Raaphorst et al. to construct arrays of strength 4. A k-cap in a projective geometry is a set of k points no three of which are collinear. In $\mathrm{PG}(3, q)$, an ovoid is a maximum-sized k-cap with $k = q^2 + 1$. Its plane sections (circles) form a 3-$(q^2 + 1, q + 1, 1)$ design, called a Möbius plane of order q. For q an odd prime power, we prove the existence of three truncated Möbius planes, such that for any choice of circles from each plane, their intersection size is at most three. From this, we construct a strength-4 covering array $\mathrm{CA}(3q^4 - 2; 4, \frac{q^2+1}{2}, q)$, for every odd prime power q. For $q \geq 11$, these covering arrays improve the size of the best-known covering arrays with the same parameters by $\sim 25\%$ [1]. These arrays can be easily constructed using linear-feedback shift-register sequences over finite fields. This is joint work with K. Shokri and B. Stevens.

References

[1] C. J. Colbourn, Covering Array Tables, available at: https://github.com/ugroempi/CAs/blob/main/ColbournTables.md, November 2024.

[2] C. J. Colbourn, C. Ingalls, J. Jedwab, M. Saaltink, K. W. Smith, and B. Stevens, Sets of mutually orthogoval projective and affine planes, *Combinatorial Theory* **1** (2024),#8.

[3] S. Raaphorst, L. Moura and B. Stevens, A construction for strength-3 covering arrays from linear feedback shift register sequences, *Designs, Codes and Cryptography* **76** (2014), 949–968.