METU Mathematics General Seminar

Numerical Semigroups: From the Frobenius Coin Problem to Hilbert Functions

Nil Şahin

Bilkent University (Ankara, Türkiye)

Abstract

The study of numerical semigroups finds its classical roots in the "Frobenius Coin Problem", famously popularized by J.J. Sylvester in the late 19th century. Sylvester's initial question "determining the largest integer that cannot be expressed as a non-negative integer linear combination of a given set of coprime integers" laid the groundwork for what is now a rich intersection of combinatorics and commutative algebra.

Through the construction of semigroup rings, these combinatorial objects provide a natural framework for studying the geometric and algebraic properties of monomial curves. In particular, the link between the numerical semigroup and the Hilbert function of its associated graded ring has become a focal point of recent research. Characterizing numerical functions that might be Hilbert functions of one-dimensional Cohen-Macaulay local rings is an open question.

There is a conjecture by Judith Sally stating that the "Hilbert function of a one-dimensional Cohen-Macaulay local ring with small enough embedding dimension is non-decreasing". In this talk, we will focus on 4-generated symmetric and pseudo-symmetric monomial curves. We will demonstrate that 4-generated pseudo-symmetric monomial curves satisfy Sally's conjecture.

Date: Thursday, 19 February 2026
Time: 15:40
Place: Gündüz İkeda Seminar Room